

# Coordinating metaheuristic agents with swarm intelligence

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**Abstract** Coordination of multi agent systems remains as a problem since there is no prominent method to completely solve this problem. Metaheuristic agents are specific implementations of multi-agent systems, which imposes working together to solve optimisation problems with metaheuristic algorithms. The idea borrowed from swarm intelligence seems working much better than those implementations suggested before. This paper reports the performance of swarms of simulated annealing agents collaborating with particle swarm optimization algorithm. The proposed approach is implemented for multidimensional knapsack problem and has resulted much better than some other works published before.

**Keywords** metaheuristic agents, · swarm intelligence, · particle swarm optimization, · simulated annealing

## 1 Introduction

Metaheuristic agents are collaborating agents to solve large scale optimisation problems in the manner of multi agent systems in which metaheuristic algorithms are adopted by the agents as the problem solvers. They are multi-agent systems identified to describe teams of search agents to operate for optimisation. This type of multi-agent systems is specific to implementations of metaheuristics to solve large scale optimisation problems (Aydin 2007). Coordination of multi agent systems remains as a problem since there is no prominent method to completely solve this problem. The-state-of-the-art of coordinating multi agents via machine learning has been extensively discussed in Panait and Luke (2005) while Vazquez-Salcada et al (2005) and Kolp et al (2006) bring forward organizational and architectural issues of multi-agent systems. Since metaheuristic agents are more specific and heavily loaded in duty, their coordination is more than those are used in

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modelling social problems. The coordination problem with metaheuristic agents constitutes of the eminent problem with metaheuristics, which is that there is no guarantee provided to find optimum solutions within a reasonable time with any metaheuristic algorithm. Instead, they usually provide with local optimum, which may not be satisfactory sometimes. One way to overcome this problem is to diversify the search conducted with the heuristics. On the other hand, distributed problem solving is mainly expected to bring more simplicity and reduction in computational time and complexity, which leads to more diversity, and more reasonable solutions. A well studied multi agent system can tackle multiple regions of the search space simultaneously. Multiple independent runs of the algorithms, which offer distributing the systems over the particular metaheuristic agents, have capabilities to carry out concurrent search within search spaces.

In this paper, the coordination problem of multi-agent systems has been tackled once again, but, with swarm intelligence algorithms this time. It is observed as expected that swarm intelligence algorithms help for better interactions and information/experience exchange. We illustrated the idea in coordinating simulated annealing agents with particle swarm optimisation algorithms implemented to solve multidimensional knapsack problem. Although there are various hybrid implementations of particle swarm optimisation and simulated annealing to solve combinatorial problems (Chan et al 2006; Dong and Qui 2006; Wang et al 2007), we have not come across with implementation of particle swarm optimisation algorithms to coordinate any metaheuristic agent such as simulated annealing agents neither any distributed versions of such hybrid algorithms. In addition, multidimensional knapsack problem has not been tackled with such hybrid algorithm either.

Previously, a couple of multi agent coordination approaches applied to metaheuristic agent teams to examine their performance in coordinating them (Aydin 2007; Hammami and Ghediera 2005). Obviously, each one provides with different benefits in tackling search and problem solving. However, swarm intelligence has not been considered for this coordination problems, whereas the notion of swarm intelligence is to substantiate artificial societies inspiring of the natural life. That is that the individuals form up a swarm are to be considered as particular agents. In contrary, the individuals remain as ordinary solutions not agents enabled with various artificial skills. In this paper, we try to prove the concept of coordinating agents with swarm intelligence algorithms.

Multidimensional knapsack problem is one of the most tackled combinatorial optimisation problems due to its flexibility in convertibility into the real world problems. The problem briefly is to maximise the total weighted p index subject to the constraints where  $x$  is a binary variable and  $r$  is a matrix of coefficients that is imposed to limit the capacities and  $b$  is the vector of upper limits.

$$\text{Maximise } \sum_{j=1}^n p_j x_j \quad (1)$$

Subject to:

$$\sum_{j=1}^n r_{ij} x_j \leq b_i \quad i = (1, \dots, m) \quad (2)$$

$$x_j \in [0, 1] \quad j = (1, \dots, n) \quad (3)$$

Equation (1) is the objective function which measures the overall capacity of the knapsacks used while Equation (2) and (3) provide the hard constraints where (2) declares the upper limit of each knapsack and (3) makes sure that the decision variable,  $x$ , can only take binary integer values. The knapsack problem has been inspired by many application areas such as networking problems, supply chain modeling problems etc. Wilbaut et al (2008) introduce a survey on the variety of knapsack problems and the ways to solve them.

The rest of the paper is organised as follows. The second section is to briefly introduce the notions of metaheuristic agents and swarm intelligence with short presentation of considered metaheuristics within the study; they are particle swarm optimisation (PSO), bee colony optimisation (BCO), and simulated annealing (SA) algorithms. The third section is to describe how to deliver the coordination of a swarm of simulated annealing agents using BCO and PSO. The experimental results are provided in section four following by the conclusions in section five.

## 2 Metaheuristic Agents and Swarm Intelligence

The concept of metaheuristic agents is identified to describe multi agent systems equipped with metaheuristics to tackle hard optimisation problems. The idea of multi agency is to build up intelligent autonomous entities whose form up teams and solve problems in harmony. The agents equipped with metaheuristics aim to solve hard and large-scale problems with their own intelligent search skills. Since standalone heuristic search usually face with local minima, ideas such as memetic algorithms, hybrid algorithms etc. have received intensive attention to overcome such shortcomings. On the other hand, the idea of multi agency eases building collaboration among various methods and approaches in a form of collaborating independent computational entities (Panait and Luke 2005; Vazquez-Salcada et al 2005; Kolp et al 2006).

Metaheuristic applications have been implemented as mostly standalone systems in an ordinary sense and examined under the circumstances of their own standalone systems. Few multi agent implementations in which metaheuristics have been exploited are examined in the literature. Various implementations of metaheuristic agents have been overviewed with respect to topologies and achievements in Aydin (2007) and Hammami and Ghediera (2005).

Swarm intelligence is referred to artificial intelligence (AI) systems where an intelligent behaviour can emerge as the outcome of the self-organisation of a collection of simple agents, organisms or individuals. Simple organisms that live in colonies; such as ants, bees, bird flocks etc. have long fascinated many people for their collective intelligence that is manifested in many of the things that they do. A population of simple units can interact with each other as well as their environment without using any set of instruction(s) to proceed, and compose a swarm intelligence system.

The swarm intelligence approaches are to reveal the collective behaviour of social insects in performing specific duties; it is about modelling the behaviour of those social insects and use these models as a basis upon which varieties of artificial entities can be developed. In such a way, the problems can be solved by models that exploit the problem solving capabilities of social insects. The motivation is to model the simple behaviours of individuals and the local interactions

with the environment and neighbouring individuals, in order to obtain more complex behaviours that can be used to solve complex problems, mostly optimisation problems (Colorno et al 1994; Kennedy and Eberhart 1995; Tasgetiren et al 2007).

## 2.1 Bee colonies

Bee colonies are rather recently developed sort of swarm intelligence algorithms, which are inspired of the social behaviour of bee colonies. This family of algorithms has been successfully used for various applications such as modelling oh communication networks (Farooq 2008), manufacturing cell formation (Pham et al 2007), training artificial neural networks (Pham et al 2006). There is a rather common opinion on that bee colony algorithms are more successful in continuous problems than combinatorial problems. The main idea behind a simple bee colony optimisation algorithm is to follow the most successful member of the colony in conducting the search. The scenario followed is that once a bee found a fruitful region, then it performs the waggle dance to communicate to the rest of the colony. Once any member of the colony realises that there is a waggle dance performance by a peer fellow, then it moves to that member's neighbourhood to collect more food. Inspiring of this natural process, bee colony optimisation algorithms are implemented for efficient search methodologies borrowing this idea to direct the search to a more fruitful region of the search space. That would result a quicker search for an appropriate solution to be considered as a neat near-optimum. For further information Pham et al (2006), (2007) and Farooq (2008) can be seen.

## 2.2 Particle swarm optimisation (PSO)

PSO is a population-based optimization technique inspired of social behaviour of bird flocking and fish schooling. PSO inventors were implementing such scenarios based on natural processes explained below to solve the optimization problems. Suppose the following scenario: a group of birds are randomly searching for food in an area, where there is only one piece of food available and none of them knows where it is, but they can estimate how far it would be. The problem here is "what is the best way to find and get that food". Obviously, the simplest strategy is to follow the bird known as the nearest one to the food. In PSO, each single solution, called a particle, is considered as a bird, the group becomes a swarm (population) and the search space is the area to explore. Each particle has a fitness value calculated by a fitness function, and a velocity of flying towards the optimum, food. All particles search across the problem space following the particle nearest to the optimum. PSO starts with initial population of solutions, which is updated iteration-by-iteration.

The pure PSO algorithm builds each particle based on, mainly, two key vectors; position  $\mathbf{x}_i$ , and velocity  $\mathbf{v}_i$ . Here,  $\mathbf{x}_i = \{x_{i1}, \dots, x_{in}\}$ , denotes the  $i^{th}$  position vector in the swarm, where  $x_{ik}$ , is the position value of the  $i^{th}$  particle with respect to the  $k^{th}$  dimension ( $k = 1, 2, 3, \dots, n$ ), while  $\mathbf{v}_i = \{v_{i,1}, \dots, v_{i,n}\}$  denotes the  $i^{th}$  velocity vector in the swarm, where  $v_{ik}$  is the velocity value of the  $i^{th}$  particle with respect to the  $k^{th}$  dimension. Initially, the position and velocity vectors are generated as continuous sets of values randomly uniformly. Personal best and global best of the

swarm are determined at each iteration following by updating the velocity and position vectors using :

$$v_{ik}(t+1) = \delta(w_t v_{ik}(t) + c_1 r_1 (y_{ik}(t) - x_{ik}(t)) + c_2 r_2 (g_k(t) - x_{ik}(t))) \quad (4)$$

where  $w$  is the inertia weight used to control the impact of the previous velocities on the current one, which is decremented by  $\beta$ , decrement factor, via  $w_{t+1} = w_t \times \beta$ ,  $\delta$  is constriction factor which keeps the effects of the randomized weight within the certain range. In addition,  $r_1$  and  $r_2$  are random numbers in  $[0,1]$  and  $c_1$  and  $c_2$  are the learning factors, which are also called social and cognitive parameters. The next step is to update the positions in the following way.

$$x_{ik}(t+1) = x_{ik}(t) + v_{ik}(t). \quad (5)$$

After getting position values updated for all particles, the corresponding solutions with their fitness values are calculated so as to start a new iteration if the predetermined stopping criterion is not satisfied. For further information, Kennedy and Eberhart (1995) and Tasgetiren et al (2007) can be seen.

PSO has initially been developed for continuous problems not for discrete ones. As MKP is a discrete problem, we use one of discrete PSO, which is proposed by Kennedy and Eberhart (1997). The idea is to create a binary position vector based on velocities as follows:

$$x_{ik}(t+1) = \frac{1}{e^{v_{ik}(t+1)}}. \quad (6)$$

where equation (5) is replaced with (6) so as to produce binary values for position vectors.

### 2.3 Simulated annealing

Simulated annealing (SA) is one of the most powerful metaheuristics used in optimisation of many combinatorial problems, which relies on a stochastic decision making process in which a control parameter called temperature is employed to evaluate the probability of moving within the neighbourhood of a particular solution. The algorithm explores across the whole search space of the problem undertaken throughout a simulated cooling process, which gradually cools a given initial hot temperature to a predefined frozen level. Given a search space  $S$ , and a particular state in search space,  $x \in S$ , a neighbourhood function,  $N(x)$ , conducts a move from  $x$ , to  $\hat{x} \in S$ , where the decision to promote the state is made subject to the following stochastic rule:-

$$x_{i+1} = \begin{cases} \hat{x}_i & \Delta x > 0 \\ \hat{x}_i e^{\frac{-\Delta x}{t_i}} \geq \rho & \\ x_i & \text{otherwise} \end{cases} \quad (7)$$

where  $\Delta x = \hat{x}_i - x_i$ ,  $i$  is the iteration index,  $\rho$  is the random number generated for making a stochastic decision for the new solution and  $t_i$  is the level of temperature (at the  $i^{th}$  iteration), which is controlled by a particular cooling schedule,  $f(t_i)$ . This means that, in order to make the new solution,  $\hat{x}_i$ , qualified for the next iteration, either the arithmetic difference,  $\Delta x$ , needs to be negative or the probability determined with  $e^{-\Delta x/t_i}$  is required to be higher than the random

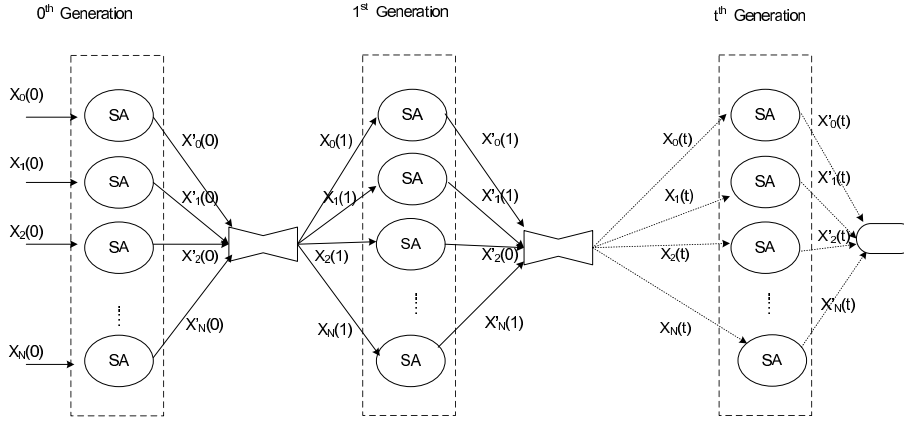
number generated,  $\rho$ , where the probability is decayed by cooling the temperature. Every state qualified to the next iteration as the consequence of the abovementioned stochastic rule gives away to a perturbation in which the solution state can be refreshed and diversified to prevent the possible local optima. A predefined number of moves attempted in this stage are repeated per iteration so as to stabilise cooling the temperature. Obviously, the stochastic rule does not allow only promoting the better solutions, but also the worse ones. However, since the probability of promoting a worse state exponentially decays towards zero, it is getting harder to exploit the perturbation facility in advanced stages of this process. That is because the temperature approaches zero as the number of iterations goes higher. More details can be found in literature such as Kolonko (1999), Aydin and Fogarty (2004) and Hammami and Ghediera (2005).

### 3 SA agents collaborating with swarm intelligence

As explained above, simulated annealing (SA) is one of the most commonly used metaheuristic approaches that offer a stochastic problem solving procedure. It is used for numerous and various successful applications (Kolonko 1999; Aydin and Fogarty 2004) in combinatorial and real optimisation domains. However, it is realised that the performance of implementations significantly depend on the neighbourhood structure as well as the hardness of the problem. In order to avoid poor performance due to such reasons, SA has been either hybridised with other peer metaheuristic algorithms such as genetic algorithm or parallelised. The main problem remains as the diversification of the search in one way or another. In this study, agents enabled with simulated annealing algorithm are used and named as SA agents.

The original idea of swarm intelligence is to form up populations of enabled individuals for collaboratively problem solving purposes. However, due to computational complexity and the hardship in furnishing the enabled individuals with multiple advanced functionalities, swarms are usually designed as population of individual static solutions evolved with various genetic and/or heuristic operators/algorithms. In this study, individuals forming up the swarms are agentified with various advance functionalities such as problem solving and communicating independently. The idea is cultivated as follows: a population of agents is created and developed with a search skill operating in the way of simulated annealing algorithm. Then, the population is organised to team up a swarm to solve the problems with their search functionalities and interaction abilities. Previously, SA agents have been organised in a variety of fashions such as with hill climbing algorithm or metropolis rule (Aydin and Fogarty 2004; Aydin 2007). The idea was to build a way of collaboration through system architecture, and gained a slight improvement in performance.

This study has aimed to find out a better way of organising agents in a more proactive collaboration so that the agents are to be enabled with contributing problem solving whilst coordinating. For this purposes, few algorithms have been examined; evolutionary simulated annealing, bee colony optimization and particle swarm optimization algorithms. Evolutionary simulated annealing is the one examined earlier for a similar purpose, to solve some other combinatorial problems (Aydin and Fogarty 2004; Yigit et al 2006; Kwan et al 2009) in which a population



**Fig. 1** The progress of evolving solutions through a typical swarm of SA agents

of solutions is created and then evolved with a fast-track simulated annealing operator on generation basis. It imposes that once an individual solution is operated by an SA, the resulting new solution is replaced with the old one. On the other hand, bee colony optimisation algorithm applies waggle dance principle of bee colonies in which the best found solution is given to every agent to kick-off a fresh search around the most promising neighbourhood. The resulted solutions are counted and sorted accordingly, and the best of them is chosen for the next generation. Ultimately, the third examined algorithm, which is found as the most promising method, is particle swarm optimization algorithm. It considers a swarm of SA agents interacting in the way of particle swarm optimisation algorithm operating.

Figure 1 sketches the progress of searching for optimum solution through generations reflecting how each agent plays its role and how the collaboration algorithm merges the intelligence produced by each agent. First of all, a swarm of SA agents is created, where each agent starts searching with a randomly generated problem state,  $x_i(0)$ . Once they finish a single run, the improved solutions,  $x'_i(0)$ , are collected into a pool and applied with a particular collaboration algorithm for exchanging information purpose. This step puts very significant impact on the speed of approximation with which the collected solutions are operated with a second algorithm to exchange information for further steps, which helps the search with diversification. There, whichever algorithm is operating will shake up and reshuffle the set of solutions, and as a result the diversifications will be re-cultivated each time. This brings an easy way of switching to different neighbourhoods within the search space. This procedure continues until a pre-defined criterion is satisfied, which is indicated in Figure 1 as the termination state of the process. The final set of results,  $x'_i(t)$ , are merged into the final pool, and a near optimum is finally determined.

The interaction of the SA agents in this way reminds the idea of variable neighbourhood search (Hansen et al 2004; Sevklı and Aydin 2006) where a systematic switch-off between search algorithms is organised in order to diversify the solutions. In an overall point of view, the swarm of SA agents sounds borrowing this idea to implement it in a wider context of exploration.

The multidimensional knapsack problem is represented in a binary way to be inline with the integer programming model in which a decision variable of  $\mathbf{x} = \{x_1, \dots, x_K\}$  plays the main role in process of optimisation, where  $\mathbf{x}$  is a vector of  $K$  binary variables. This is also the way how to present a problem state. Here, once a corresponding amount is decided to be included in knapsack  $k$ , then  $x_k$  becomes 1 otherwise 0. The heuristic search for optimum value is conducted via use of neighbourhood structure of inverter function, which simply inverts the value of a randomly selected variable at a time. The main search is conducted by a so-called fast-track SA algorithm embedded in each agent with inverting values of up to 3 variables at a time. A complete search operation by a SA agent is measured based a cost/fitness function, which relates each state of the problem to a corresponding real value.

$$f_i : \mathbf{x}_i(\mathbf{t}) \rightarrow \mathbb{R} \quad (8)$$

where  $\mathbf{x}_i$  is the  $i^{th}$  vector of decision variables within the swarm, which corresponds to the  $i^{th}$  SA agent. In the case of multidimensional knapsack problem, the fitness/cost function,  $f_i$ , corresponds to the objective function (Equation (1)). An agent embedded with fast-track SA explores for better state of the problem taking  $\mathbf{x}_i^h = \mathbf{x}_i(t)$  and producing  $\mathbf{x}_i^f = \mathbf{x}'_i(t)$  following the main procedure of SA algorithm,

$$\mathbf{x}_i^f = SA_i(\mathbf{x}_i^h) \quad (9)$$

where  $i$  is the index for agents,  $h$  and  $f$  represent "hot" and "frozen" keywords<sup>1</sup> and  $SA_i(\cdot)$  is the problem solving process of the  $i^{th}$  agent. There, the improvement towards the optimum value is measured as  $f_{hot}$  to  $f_{frozen}$ . As expected, the overall search by the whole swarm of SA agents is conducted generation-by-generation as is done in other evolutionary methodologies. Hence, implementing these multiple SA agents, there will be  $N$  number of initial states of the problem considered by  $N$  agents and  $N$  number of improved results produced per generation. The whole swarm will include a set of fitness values representing the state of the swarm with respect to the solution quality.  $\mathbf{F}(\mathbf{t}) = \{f_0, \dots, f_K\}$  is the fitness vector of generation  $t$  through the overall problem solving process. The swarm of SA agents will find the best of the generation,  $\mathbf{x}^b(t)$ , based on the fitness vector, which provides  $f_{best}$ . Moving to the next generation is subject to the level of satisfaction with the solution quality. If it is not sufficiently optimised, yet, the next generation will be gone through the determination of new set of hot solutions, where a coordination algorithm is needed to combine all the experiences of the agents, and let them select their new hot states. As explained before, the coordination algorithms considered in this research are evolutionary simulated annealing (ESA), bee colony optimisation (BCO) and particle swarm optimisation (PSO). ESA imposes each agent to take up  $\mathbf{x}_i^f(t)$  as  $\mathbf{x}_i^h(t+1)$ , where  $t$  is the index for generations, while BCO imposes  $\mathbf{x}^b(t)$  to every agents to kick off search for next generation. PSO runs the usual interaction procedure, which explained above, to determine the new hot solutions. Therefore, a new hot solution will be produced as the result of  $\mathbf{x}_i^h(t+1) = \mathbf{pso}_t(\mathbf{x}_i^f, \mathbf{x}_i^{pb}, \mathbf{x}^b)$ , where  $\mathbf{x}_i^{pb}$  and  $\mathbf{x}^b$  are personal and global best solutions. The whole procedure of coordination by PSO lasts between  $\mathbf{pso}_0(\cdot)$  and  $\mathbf{pso}_T(\cdot)$ , where  $T$  is the final generation through the whole process.

<sup>1</sup> "Hot" and "frozen" are two preferred keywords to express the "initial" and "final", respectively, in order to be inline with the jargon used in simulated annealing studies.



#### 4 Experimental Study

This experimental study is not especially to solve multidimensional knapsack problem (MKP), but to test the performance of various approaches including swarm intelligence to coordinate metaheuristic agents. The abovementioned swarm intelligence model for SA agents has been examined with solving multidimensional knapsack problem, which is one of well-known NP-Hard combinatorial optimization problems. For this purpose, a swarm of SA agents, each was configured with a fast-track SA procedure, was created. Three approaches are examined for the purpose of an efficient coordination: an evolutionary simulated annealing (ESA) algorithm (Aydin and Fogarty 2004), a bee colony optimisation (BCO) algorithm (Pham et al; 2006;2007), and a binary represented PSO algorithm (Kennedy and Eberhart 1997), were implemented to work as a coordinator algorithm. The multidimensional knapsack problem was represented with a binary coding scheme.

SA procedure to be run by each agent was investigated for whether to be a 100 iteration long SA to run through 300 generations or a 200 iteration long SA to run 300 generations. The preliminary results confirmed that a 200 iteration long SA algorithm with varying number of generations (Aydin 2008). That was inline with previous researches. In addition, the size of swarm was investigated in a range of 5 to 50. The experimentation is conducted with only two moderately hard MKP benchmarks, namely MKP6 and MKP7 collected from OR library (Beasley 1990). The results are summarised in Table 1, 2 and 3 with the solution quality and computational time, where the solution quality is measured with relative percentage of error (RPE).

$$RPE = \frac{f_{opt} - f_{avg}}{f_{opt}} \quad (10)$$

where  $f_{opt}$  and  $f_{avg}$  are the optimum and the average values of experimented results. The average value,  $f_{avg}$ , is the mean calculated over 50 replications. The second performance measure is the averaged CPU time, which is the mean of the 50 replications. The performance with respect to the solution quality is primarily considered and the one with respect to CPU is secondarily considered in case of any tight comparisons.

The implementation of the systems has been done using POP C++, which is a GRID programming language developed by Nguyen and Kuonen (2007). It is such a unique distributed programming language that uses object distribution over the targeted infrastructure, and arrange automatic communications among the distributed entities. This property of POP C++ eases its use in development of multi agent systems. All experiments were conducted on GRID infrastructure in Computer Science department of Applied University of Western Switzerland in Fribourg.

Table 1 presents experimental results with the most fast-track SA agents coordinated with all three approaches against various swarm sizes. The SA algorithm is configured to run 200 iterations without any inner replications, which means that the cooling schedule allows operating once per level of temperature. All three algorithms, ESA, BCO and PSO, are separately applied to the same swarm of SA agents under the same circumstances. The swarm size varies between 5 and 50 agents. The multidimensional knapsack benchmark problems tackled are MKP6 and MKP7 in all cases. All experiments are replicated for 50 times. The worst level of achievement with respect to quality of solution is delivered by BCO while

**Table 1** Experimental results of the swarm of fast-track SA agents with single inner iteration and coordinated with various approaches

Swarm Size		ESA		BCO		PSO	
		RPE	CPU	RPE	CPU	RPE	CPU
MKP6	5	0.03495	0.11	0.02808	0.73	0.00257	0.84
	10	0.01183	0.43	0.02021	1.29	0.00214	1.38
	15	0.00899	0.86	0.01694	1.73	0.00170	2.31
	20	0.01052	1.08	0.01530	2.25	0.00203	2.40
	30	0.00762	1.80	0.01344	2.79	0.00098	2.56
	40	0.00768	1.86	0.01226	4.34	0.00122	3.67
	50	0.00633	2.56	0.01093	5.28	0.00061	4.09
MKP7	5	0.03748	0.14	0.04077	0.59	0.00307	0.78
	10	0.02170	0.52	0.03270	1.18	0.00175	1.30
	15	0.01528	1.01	0.02782	1.57	0.00112	1.31
	20	0.01407	1.17	0.01906	2.10	0.00064	1.31
	30	0.00961	2.34	0.01516	2.95	0.00014	0.93
	40	0.00821	2.20	0.01736	4.35	0.00030	1.21
	50	0.00865	2.66	0.01979	5.38	0.00028	1.15

PSO has the best and ESA has an intermediate level of achievement. On the other hand, the shortest computational time achieved by ESA while the longest one is done by BCO and PSO is in the middle. The overall gain by PSO over BCO, which is the worst case, remain between 90-95% and 25-33% by ESA. The time-wise gain is 49% and 31% by ESA and PSO, respectively. The swarm-size-wise performance is a significant too. For both benchmarks, the size of the swarm indicates a gradual increase in performance in all cases; the solution quality index linearly decreases. Another most interesting fact is that the error level indicated by PSO is nearly about 10% of both ESA's and BCO's levels.

Table 2 presents the results of experimentations sets which considered 5 inner iterations per SA cycle. These results are much better ones comparing to the single inner iteration case. All three algorithms that coordinate fast-track SA agents, with 5 inner iterations per cycle this time, and improve their performance gradually through the growing size of the swarm. ESA hits 100% achievement with 30 and 40-agent swarms, while PSO hits about 99% in both cases. BCO remains improving in comparison with the single inner case, but outperformed by both ESA and PSO. The overall gain by PSO over BCO, which is the worst case remain between 65-95% and 84-95% by ESA. The gain with respect to CPU times is 82% and 39% by ESA and PSO, respectively.

Table 3 shows the experimental results of more focused SA agents, which are replicating 10 times per step of cooling schedule. Since this way of search is more focused, the results of both ESA and PSO hit the optimum 100% with swarm size of 20. Therefore, the experimentation has not proceeded further. As the table manifests, PSO and ESA compete each other, but outperform BCO with respect to both quality of solution and computational time, where the gain over BCO in terms of solution quality is 82-89% and 82-92% by ESA and PSO, respectively. The achievement via CPU time is 64% and 22% by ESA and PSO, respectively.

Fig. 2 indicates the averaged-RPE results of each coordinating approach per benchmark per level of inner iterations in fast-track SA agents. The averaged results are tabulated across horizontal axis pointing out the overall achievement

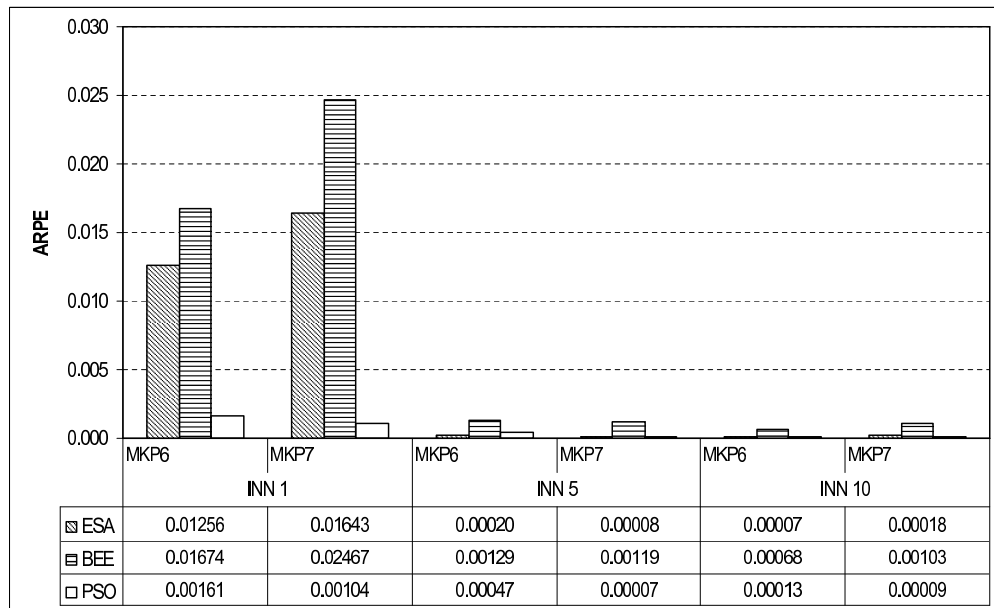
**Table 2** Experimental results of swarm of fast-track SA agents with 5 inner iterations and coordinated with various approaches

Swarm Size		ESA		BCO		PSO	
		RPE	CPU	RPE	CPU	RPE	CPU
MKP6	5	0.00069	0.03	0.00182	0.64	0.00076	0.70
	10	0.00031	0.34	0.00139	1.21	0.00066	1.07
	15	0.00013	0.32	0.00143	1.65	0.00068	1.81
	20	0.00005	0.29	0.00100	1.64	0.00042	1.33
	30	0.00000	0.27	0.00090	1.91	0.00021	1.08
	40	0.00000	0.20	0.00121	2.73	0.00011	1.42
MKP7	5	0.00031	0.08	0.00190	0.56	0.00013	0.24
	10	0.00009	0.30	0.00128	0.92	0.00004	0.26
	15	0.00006	0.32	0.00118	1.15	0.00009	0.51
	20	0.00003	0.28	0.00120	1.27	0.00009	0.65
	30	0.00000	0.25	0.00078	1.37	0.00002	0.57
	40	0.00000	0.28	0.00082	1.59	0.00002	0.44

**Table 3** Experimental results of ESA agents with 10 inner iterations and coordinated with various approaches

Swarm Size		ESA		BCO		PSO	
		RPE	CPU	RPE	CPU	RPE	CPU
MKP6	5	0.00027	0.09	0.00086	0.44	0.00029	0.48
	10	0.00002	0.17	0.00063	0.66	0.00013	0.66
	15	0.00000	0.14	0.00066	0.80	0.00008	0.49
	20	0.00000	0.14	0.00060	0.97	0.00000	0.33
MKP7	5	0.00072	0.16	0.00141	0.45	0.00019	0.35
	10	0.00000	0.13	0.00130	0.62	0.00013	0.55
	15	0.00000	0.13	0.00070	0.64	0.00002	0.44
	20	0.00000	0.14	0.00073	0.75	0.00000	0.48

of each approach, where the benchmark problems are indicated as MPK6 and MPK7 with each inner iteration case. INN 1, INN 5 and INN 10 indicate the inner iteration level of 1, 5 and 10. As both the graph and the tabulated values reveal, the performance of ESA and PSO comparable beyond the inner iterations of 5 onward. However, their achievements remain significantly different in the case of inner iteration 1, which is the simplest form of cooling process in SA procedure. PSO clearly and significantly outperform both ESA and BCO approaches, while ESA does better than BCO. Depending on their level of difficulty, simulated annealing algorithms are configured with the level of inner iterations, whereas some problems favour of higher level of inner iterations, but some do not do at all, especially those are time sensitive such as resource scheduling problem of radio access networks (Kwan et al 2009), where the speed of the algorithms are measured in nano-second level. Therefore, more focused and intensified search will not help solving such problems at all.



**Fig. 2** Average performance of agent swarms in various sizes operating with all three algorithms and all three inner-iteration levels

## 5 Conclusions

Metaheuristic agent swarms need collaboration in one way or another to deliver an efficient problem solving services. In this paper, three collaboration algorithms have been examined with respect to efficiency in solution quality. The agents form up the swarms, which are configured as simulated annealing agents to solve multidimensional knapsack problem. Evolutionary simulated annealing, bee colony optimisation and particle swarm optimisation algorithms are used for collaboration purposes. The algorithm found best to be paired with SA agents is PSO, which is a relatively newer swarm intelligence approach that has good record for continuous problems, but usually needs a local search embedded in for combinatorial problems. On the other hand SA needs to incorporate with other search methods for diversification. It is significantly concluded that collaborating metaheuristic agents with swarm intelligence algorithm adds up value into the quality of solution. This incorporation works in the form of a variable search algorithm in an overall point of view. It also keeps the properties of ESA (Yigit et al 2006) as it reheats the temperature, and works with a population.

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